اسم المادة الدراسية: الميكانيك وخواص المادة

عدد الساعات النظرية: 2

عدد الساعات العملية : 2

عدد الوحدات السنوية: 6

المرحلـــــــــة : الأولى

المنهاج السنوي

1. المتجهات
2. داينميك الجسيم (قوانين نيوتن للحركة ، الزخم الخطي ، القوة كدالـــة للموضـــع ، القوة كدالة للسرعة ، القوة كدالة للزمن ، الحركة التوافقية ، الحركـــة التوافقيـــة المتضائلة ، الحركة التوافقية الاضطرارية ، قـــاعدة الشـــغل ، القـــوى المحافظـــة ومجالات القوة ، دالة الطاقة الكامنة ، مؤثر دلتا ، حركة الجسيمات المشحونة فـــي المجالات الكهربائية والمغناطيسية ، الحركة على خط منحني ، البندول البسيط)
3. حركة المحاور المرجعية (حركة المحاور الانتقالية ، الحركـــة العامـــة للمحـــاور ، داينميك الجسيم في المحاور الدوارة ، تأثيرات دوران الأرض)
4. داينميك منظومة الجسيمات (مركز الكتلة والزخم الخطي ، الزخم الزاوي والطاقـــة الحركية لمنظومة الجسيمات ، حركة جســـمين والكتلـــة المصغـــرة ، التصادمـــات بأنواعها ، مقارنة بين المحاور المختبرية ومحاور مركز الكتلة)
5. ميكانيك الجسم الصلد (مركز الكتلة لجسم صلد ، دوران جسم صلد حـــول محـــور ثابت ، عزم القصور الذاتي ، حساب عزم القصور الذاتي ، زخـــم الجســـم الصلـــد الزاوي ، الطاقة الحركية الدورانية لجسم صلد)
6. النظرية النسبية الخاصة (تجربة مايكلسن – مورلي ، فرضيات انشتين في النسبية الخاصة ، تحويلات لورنتز ونتائجها ، تقلص الطول وتمديد الزمن ، الفضاء والزمن ، تحويلات السرع ، تغير الكتلة مع السرعة ، علاقة الكتلة والطاقة) .

اسم الكتاب المقترح : David Halliday and Robert Resnick , 1988

Physics , H.C. Ohanian , 1985 .

المناقشة : يترك تحديد ساعات المناقشة لمجلس القسم .

References:

(1) physics : by Alenso & Finn 6-printing, 1981

(2) University physics : by Francis and others, 1982

(3) principle of physics : by Jerry B. Marion and William F. Hornyak, 1984

(4) اساسيات الفيزياء : ترجمة ف. بوش و جيرد .

(5) الفيزياء الكلاسيكيه ، تأليف فريدريك بوش وآيوجين 2008
الطبعه العربيه الثالثه / الدار الدوليه - مصر .

معلومات مفيدة

① الكمية الفيزيائية : المسافة (Distance) الوحدة الأساسية المتر m.

$1\ Fermi = 10^{-15} m$ فيرمي

$1\ Angstrom = 10^{-10} m$ انكستروم A°

$1\ nano\ meter = 10^{-9} m$ نانومتر nm

$1\ micrometer = 10^{-6} m$ مايكرومتر μm

$1\ millimeter = 1 mm = 10^{-3} m$ مليمتر

$1\ Centemeter = 1 cm = 10^{-2} m$ سنتيمتر

$1\ Kilometer = 1 Km = 10^{3}$ كيلومتر

② الكتلة : Mass الوحدة الأساسية Kg.

$1\ microgram = 1 \mu g = 10^{-6} gm = 10^{-9} Kg$ مايكروغرام

$1\ milligram = 1 mg = 10^{-3} gm = 10^{-6} Kg$ مليغرام

$1\ gram = 1 gm = 10^{-3} Kg$ غرام

$1\ ton = 10^{6} gm = 10^{3} Kg$ 1 طن

③ الزمن : Time الوحدة الأساسية الثانية sec.

$1\ pico\ second = 1 ps = 10^{-12} sec$ بيكوثانية

$1\ nano\ second = ns = 10^{-9} sec$ نانوثانية

$1\ micro\ second = 1 \mu s = 10^{-6} sec =$ مايكروثانية

$1\ millisecond = 1 ms = 10^{-3} sec.$ مللي ثانية

$1\ minute = 1 min = 60 sec$ 1 دقيقة

$1\ hour = 1 hr = 3600 sec$ 1 ساعة

Measurement and Units

meter: Unit of length ($1\ m = 10^{-3}\ Km = 100\ cm = 1000\ mm$)

Kilogram: Unit of mass ($1\ kg = 1000\ gm$)

atomic mass unit: $1\ amu = \frac{1}{12}$ of the mass of a ^{12}C atom

$$\Rightarrow 1\ amu = \frac{1}{12} * 1.9925 * 10^{-26}\ Kg$$

$$= 1.6604 * 10^{-27}\ Kg$$

second: Unit of time (s), $1\ hr = 60 min * 60 \frac{sec}{min} = 3600 sec$

Coulomb: Unit of electric charge (c)

e = electron or Proton charge = $1.6 * 10^{-19}$ c.

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* Prefixes for Powers of ten:

magnitude	Prefix	symbol
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	Pico	P
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{3}	Kilo	K
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T

- **Density**: Defined, mass per unit volume:

$$\rho = \frac{mass}{Volume} = \frac{m}{V} = Kg \cdot m^{-3} \text{ or } gm \cdot cm^{-3}$$

- let ρ_1 and ρ_2 are, The densities of two different substances, their relative density is the ratio:

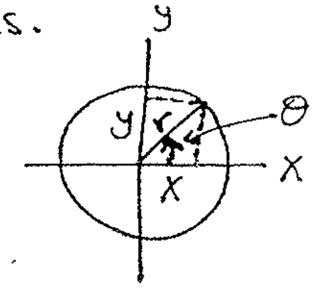
$$\rho_{21} = \frac{\rho_2}{\rho_1}$$

$$1\ Kg \cdot m^{-3} = 1\ \frac{\cancel{Kg}}{\cancel{m^3}} * \frac{1000\ \frac{gm}{\cancel{Kg}}}{\left(10^2\ \frac{Cm}{\cancel{m}}\right)^3} = \frac{1000}{10^6}\ gm \cdot Cm^{-3}$$

$$\therefore \boxed{1\ Kg \cdot m^{-3} = 10^{-3}\ gm \cdot cm^{-3}}$$

- **Plane Angles**: There are two systems for measuring plane angles: Degree and radians.

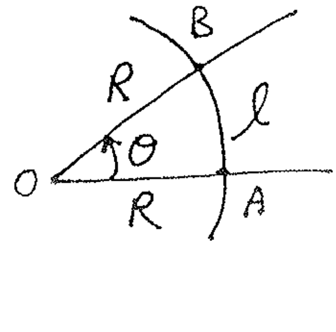
① Degree system: θ°



$$\tan\theta = \frac{y}{x}, \quad \sin\theta = \frac{y}{r}, \quad \cos\theta = \frac{x}{r}$$

$$r = \sqrt{x^2 + y^2}, \quad \sin^2\theta + \cos^2\theta = 1$$

② radian system:



$$\theta_{(rad)} = \frac{l}{R}$$

$$\pi(rad) = 180^\circ \Rightarrow 2\pi = 360^\circ$$

$$1^\circ = \frac{\pi}{180}\ rad = 0.0174\ rad, \quad 1\ rad = \frac{180^\circ}{\pi} = 57^\circ\ 17'\ 44.9''$$

Time:

1 s = 1.667×10^{-2} min = 2.778×10^{-4} hr
 = 3.169×10^{-8} yr
1 min = 60 s = 1.667×10^{-2} hr
 = 1.901×10^{-6} yr
1 hr = 3600 s = 60 min = 1.141×10^{-4} yr
1 yr = 3.156×10^{7} s = 5.259×10^{5} min
 = 8.766×10^{3} hr

Length:

1 m = 10^{2} cm = 39.37 in. = 6.214×10^{-4} mi
1 mi = 5280 ft = 1.609 km
1 in. = 2.540 cm
1 Å (angstrom) = 10^{-8} cm = 10^{-10} m
 = 10^{-4} μ (micron)
1 μ (micron) = 10^{-6} m
1 AU (astronomical unit) = 1.496×10^{11} m
1 light year = 9.46×10^{15} m
1 parsec = 3.084×10^{16} m

Angle:

1 radian = 57.3°
1° = 1.74×10^{-2} rad
1′ = 2.91×10^{-4} rad
1″ = 4.85×10^{-6} rad

Area:

1 m^2 = 10^{4} cm^2 = 1.55×10^{-3} in^2
 = 10.76 ft^2
1 in^2 = 6.452 cm^2
1 ft^2 = 144 in^2 = 9.29×10^{-2} m^2

Volume:

1 m^3 = 10^{6} cm^3 = 10^{3} liters
 = 35.3 ft^3 = 6.1×10^{4} in^3
1 ft^3 = 2.83×10^{-2} m^3 = 28.32 liters
1 in^3 = 16.39 cm^3

Velocity:

1 m s^{-1} = 10^{2} cm s^{-1} = 3.281 ft s^{-1}
1 ft s^{-1} = 30.48 cm s^{-1}
1 mi min^{-1} = 60 mi hr^{-1} = 88 ft s^{-1}

Acceleration:

1 m s^{-2} = 10^{2} cm s^{-2} = 3.281 ft s^{-2}
1 ft s^{-2} = 30.48 cm s^{-2}

Mass:

1 kg = 10^{3} g = 2.205 lb
1 lb = 453.6 g = 0.4536 kg
1 amu = 1.6604×10^{-27} kg

Force:

1 N = 10^{5} dyn = 0.2248 lbf = 0.102 kgf
1 dyn = 10^{-5} N = 2.248×10^{-6} lbf
1 lbf = 4.448 N = 4.448×10^{5} dyn
1 kgf = 9.81 N

Pressure:

1 N m^{-2} = 9.265×10^{-6} atm
 = 1.450×10^{-4} lbf in^{-2}
 = 10 dyn cm^{-2}
1 atm = 14.7 lbf in^{-2} = 1.013×10^{5} N m^{-2}
1 bar = 10^{6} dyn cm^{-2}

Energy:

1 J = 10^{7} ergs = 0.239 cal
 = 6.242×10^{18} eV
1 eV = 10^{-6} MeV = 1.60×10^{-12} erg
 = 1.07×10^{-9} amu
1 cal = 4.186 J = 2.613×10^{19} eV
 = 2.807×10^{10} amu
1 amu = 1.492×10^{-10} J
 = 3.564×10^{-11} cal = 931.0 MeV

Temperature:

K = 273.1 + °C
°C = $\frac{5}{9}$ (°F − 32)
°F = $\frac{9}{5}$ °C + 32

Power:

1 W = 1.341×10^{-3} hp
1 hp = 745.7 W

Electric Charge:*

1 C = 3×10^{9} stC
1 stC = $\frac{1}{3} \times 10^{-9}$ C

Current:*

1 A = 3×10^{9} stA
1 stA = $\frac{1}{3} \times 10^{-9}$ A
1 μA = 10^{-6} A, 1 mA = 10^{-3} A

Electric Field:*

1 N C^{-1} = 1 V m^{-1} = 10^{-2} V cm^{-1}
 = $\frac{1}{3} \times 10^{-4}$ stV cm^{-1}

Electric Potential:*

1 V = $\frac{1}{3} \times 10^{-2}$ stV
1 stV = 3×10^{2} V

Resistance:

1 Ω = 10^{6} μΩ
1 MΩ = 10^{6} Ω

Capacitance:*

1 F = 9×10^{11} stF
1 stF = $\frac{1}{9} \times 10^{-11}$ F
1 μF = 10^{-6} F, 1 pF = 10^{-12} F

Magnetic Field:

1 T = 10^{4} gauss, 1 gauss = 10^{-4} T

Magnetic Flux:

1 Wb = 10^{8} maxwell, 1 maxwell = 10^{-8} Wb

Magnetizing Field:

1 A m^{-1} = $4\pi \times 10^{-3}$ oersted
1 oersted = $1/4\pi \times 10^{3}$ A m^{-1}

* In all cases, 3 actually means 2.998 and 9 means 8.987.

Table A-2 Fundamental Constants	Constant	Symbol	Value
	Velocity of light	c	2.9979×10^{8} m s^{-1}
	Elementary charge	e	1.6021×10^{-19} C
	Electron rest mass	m_e	9.1091×10^{-31} kg
	Proton rest mass	m_p	1.6725×10^{-27} kg
	Neutron rest mass	m_n	1.6748×10^{-27} kg
	Planck constant	h	6.6256×10^{-34} J s
		$\hbar = h/2\pi$	1.0545×10^{-34} J s
	Charge-to-mass ratio for electron	e/m_e	1.7588×10^{11} kg^{-1} C
	Quantum charge ratio	h/e	4.1356×10^{-15} J s C^{-1}
	Bohr radius	a_0	5.2917×10^{-11} m
	Compton wavelength:		
	of electron	$\lambda_{C,e}$	2.4262×10^{-12} m
	of proton	$\lambda_{C,p}$	1.3214×10^{-15} m
	Rydberg constant	R	1.0974×10^{7} m^{-1}

Constant	Symbol	Value
Bohr magneton	μ_B	9.2732×10^{-24} J T^{-1}
Avogadro constant	N_A	6.0225×10^{23} mol^{-1}
Boltzmann constant	k	1.3805×10^{-23} J K^{-1}
Gas constant	R	8.3143 J K^{-1} mol^{-1}
Ideal gas normal volume (STP)	V_0	2.2414×10^{-2} m^3 mol^{-1}
Faraday constant	F	9.6487×10^{4} C mol^{-1}
Coulomb constant	K_e	8.9874×10^{9} N m^2 C^{-2}
Vacuum permittivity	ϵ_0	8.8544×10^{-12} N^{-1} m^{-2} C^2
Magnetic constant	K_m	1.0000×10^{-7} m kg C^{-2}
Vacuum permeability	μ_0	1.2566×10^{-6} m kg C^{-2}
Gravitational constant	γ	6.670×10^{-11} N m^2 kg^{-2}
Acceleration of gravity at sea level and at equator	g	9.7805 m s^{-2}

Numerical constants: $\pi = 3.1416$; $e = 2.7183$; $\sqrt{2} = 1.4142$; $\sqrt{3} = 1.7320$

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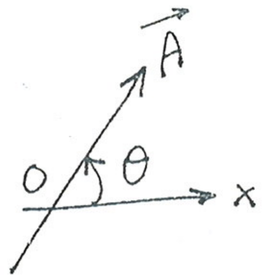
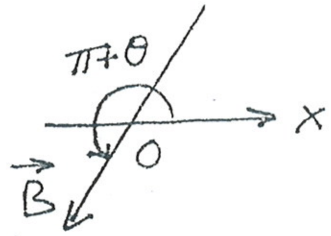
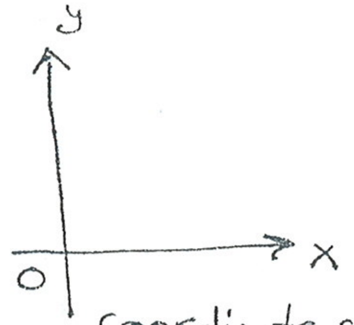
Chapter 1 : Vectors



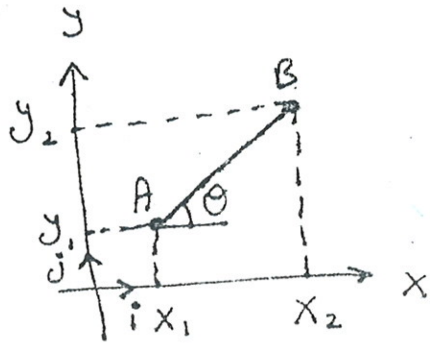




coordinate axes

* In plane, Opposite direction are defined by angles θ and $(\pi+\theta)$

* $\overrightarrow{AB} = (x_2 - x_1)\, i + (y_2 - y_1)\, j = xi + yj$



i, j are Units vector, for x and y axes.

$\overrightarrow{BA} = -\overrightarrow{AB}$

* <u>Scalars</u> : Physical quantities are completely determined by Their magnitude. for example : Volume, temperature, time, mass, charge, energy

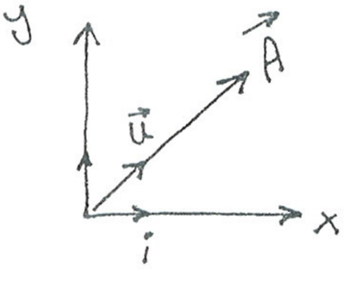
* <u>Vectors</u> : physical quantities require, for their complete determination, a direction in addition to their magnitude. for example displacement, Force, moment, Torque, velocity

* <u>Unit vector</u> : is a vector whose magnitude is one.

$\vec{u}$ Parallel to the vector;

* $|\vec{A}|$ is the magnitude of $\vec{A}$

$\Rightarrow |\vec{A}| = \sqrt{x^2+y^2}$



$\therefore \vec{A} = \vec{u}\,|\vec{A}|$ or $\boxed{\vec{u} = \dfrac{\vec{A}}{|\vec{A}|}}$ $\Rightarrow |\vec{u}| = 1$

also $|$any unit vector$| = 1$

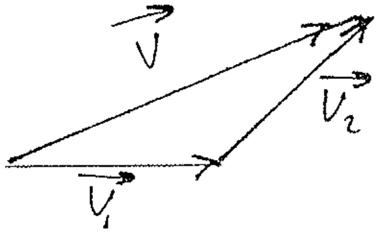
__example__: Find the Vector $\vec{AB}$ from two points $A(2,1)$, $B(-1,2)$ also; Find Unit vector in direction of $\vec{AB}$.

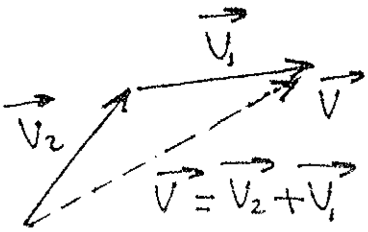
__sol.__ $\vec{AB} = (-1-2)i + (2-1)j = -3i + j$

Unit vector $\vec{u} = \dfrac{\vec{AB}}{|\vec{AB}|} = \dfrac{-3i+j}{\sqrt{(-3)^2+(1)^2}} = \dfrac{1}{\sqrt{10}}(-3i+j)$

__Addition of Vectors:__

$\vec{V} = \vec{V_1} + \vec{V_2}$

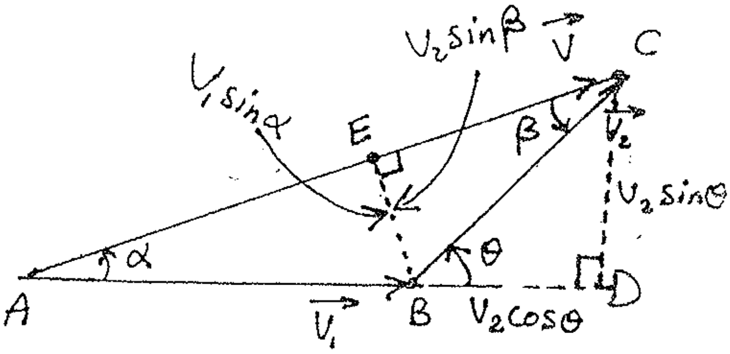




* To compute the magnitude of $\vec{V}$ we use the relation:

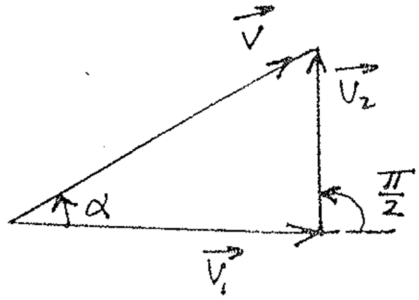
$$\boxed{V = |\vec{V}| = \sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos\theta}}$$

θ is the angle between $\vec{V_1}$ and $\vec{V_2}$.



* To determine the direction of $\vec{V}$ we need to know the angle α or β, where:

$$\boxed{\frac{V}{\sin\theta} = \frac{V_1}{\sin\beta} = \frac{V_2}{\sin\alpha}}$$



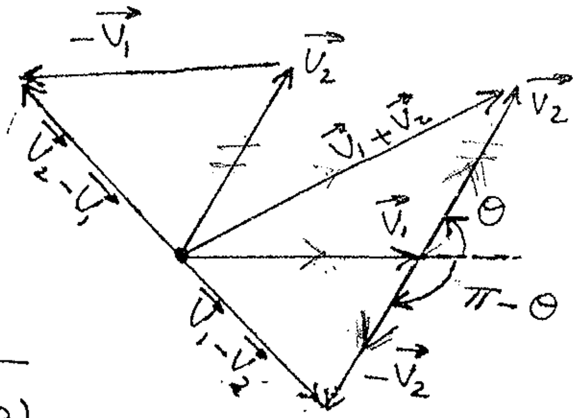
* In special case, when V_1 and V_2 are perpendicular $(\theta = \frac{\pi}{2})$

$V = \sqrt{V_1^2 + V_2^2}$ and $\tan\alpha = \dfrac{V_2}{V_1}$

* Difference between two vectors:

$\vec{D} = \vec{V_1} - \vec{V_2} = \vec{V_1} + (-\vec{V_2})$

$\pi - \theta$: is the Angle between the vector $\vec{V_1}$ and $(-\vec{V_2})$



The magnitude of the difference is:

$$D = \sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos(\pi - \theta)}$$

since $\cos(\pi-\theta) = \overset{-1}{\cos\pi}\cdot\cos\theta - \overset{0}{\sin\pi}\cdot\sin\theta = -\cos\theta$

$$\therefore D = \sqrt{V_1^2 + V_2^2 - 2V_1V_2\cos\theta}$$

---

* Proof the relation $V = \sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos\theta}$

From the fig.: $(AC)^2 = (AD)^2 + (DC)^2$ (Page 3-2)

But $AD = AB + BD = V_1 + V_2\cos\theta$

and $DC = V_2\sin\theta$

Therfore: $V^2 = (V_1 + V_2\cos\theta)^2 + (V_2\sin\theta)^2$

$$V^2 = V_1^2 + V_2^2\cos^2\theta + 2V_1V_2\cos\theta + V_2^2\sin^2\theta$$

$$V^2 = V_1^2 + V_2^2(\cos^2\theta + \sin^2\theta) + 2V_1V_2\cos\theta \quad ; \cos^2\theta + \sin^2\theta = 1$$

$$\therefore V = \sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos\theta}$$

* Proof the relation: $\frac{V}{\sin\theta} = \frac{V_1}{\sin\beta} = \frac{V_2}{\sin\alpha}$

from the fig. In Page 3-2

$CD = AC \sin\alpha$ (from triangle ACD)

$CD = BC \sin\theta$ (from triangle BDC) ; $AC = V$; $BC = V_2$

$\Rightarrow$ $V \sin\alpha = V_2 \sin\theta$ or $\boxed{\frac{V}{\sin\theta} = \frac{V_2}{\sin\alpha}}$

Similarly: $BE = V_1 \sin\alpha = V_2 \sin\beta$

$\Rightarrow \boxed{\frac{V_2}{\sin\alpha} = \frac{V_1}{\sin\beta}}$

Combining both results, we have the symmetrical relation:

$$\boxed{\frac{V}{\sin\theta} = \frac{V_1}{\sin\beta} = \frac{V_2}{\sin\alpha}}$$

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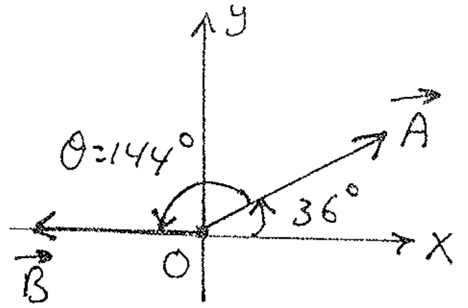
ex: Given two vectors: $\vec{A}$ is 6 units long and makes an angle of $+36^\circ$ with the positive x-axis; $\vec{B}$ is 7 units long and is in the direction of the negative x-axis. Find: a- the sum $\vec{A}+\vec{B}$ of the two vectors; b- the difference $\vec{A}-\vec{B}$ between the vectors.

Sol: Draw the vectors on a set of coordinate:



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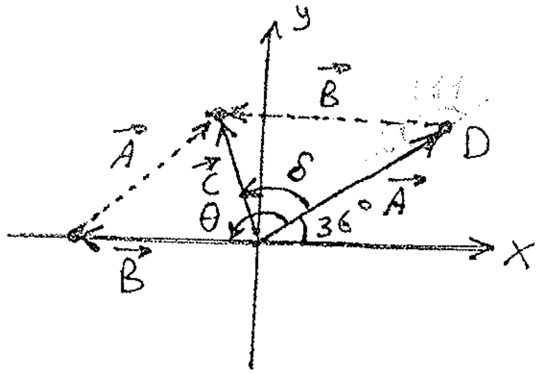
(a) The sum: $\vec{C} = \vec{A} + \vec{B}$

$C = \sqrt{36+49+2(6)(7)\cos 144}$

$C = 4.128$ Units



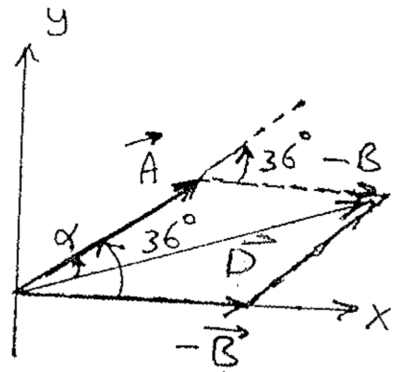
$$\frac{C}{\sin\theta} = \frac{B}{\sin\delta}$$

so that $\Rightarrow \sin\delta = \frac{B\sin 144°}{C} = 0.996 \Rightarrow \delta = 85°$

The direction of C with Positive x-axis: $36° + 85° = 121°$

(b) The difference: $\vec{D} = \vec{A} - \vec{B}$

$\vec{D} = \vec{A} + (-\vec{B})$



either: $D = \sqrt{36+49-2(6)(7)\cos 144}$

$= 12.31$ Units

or: $D = \sqrt{36+49+2(6)(7)\cos 36} = 12.31$ Units

To find the direction of $\vec{D}$:

$$\frac{D}{\sin 36°} = \frac{B}{\sin\alpha} \Rightarrow \sin\alpha = \frac{B\sin 36°}{D} = 0.334 \text{ or } \alpha = 19.5°$$

and $\vec{D}$ with x^{+}-axis is: $36° - 19.5° = 16.5°$

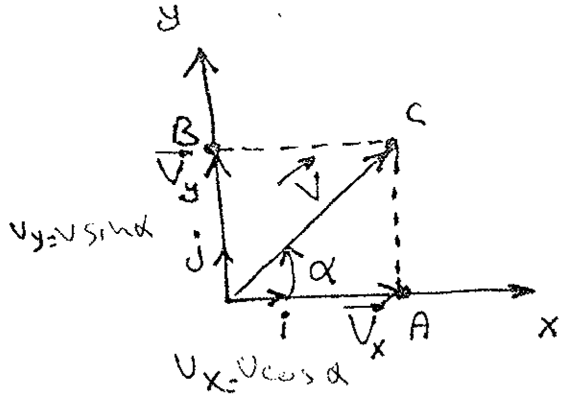
* Components of A vector: طريقة التحليل (التعامد)

$\vec{V} = \vec{V}_x + \vec{V}_y$ (In plane x-y)

$V_x = V\cos\alpha$, $V_y = V\sin\alpha$

$\vec{V}_x = i\,V_x$, $\vec{V}_y = j\,V_y$

$\Rightarrow \boxed{\vec{V} = i\,V_x + j\,V_y}$



$\tan\alpha = \dfrac{V_y}{V_x}$

$\Rightarrow \vec{V} = i\,V\cos\alpha + j\,V\sin\alpha \Rightarrow \vec{V} = V(i\cos\alpha + j\sin\alpha)$

$\boxed{\vec{u} = \dfrac{\vec{V}}{V} = i\cos\alpha + j\sin\alpha}$

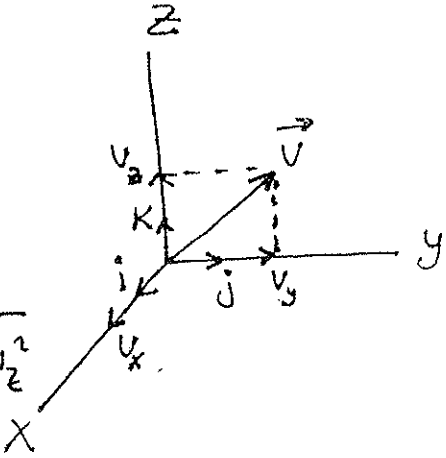
* Three rectangular components in space:

$\vec{V} = i V_x + j V_y + K V_z$

K: Unit vector for z-axis

The magnitude of vector in space:

$V^2 = V_x^2 + V_y^2 + V_z^2 \Rightarrow V = \sqrt{V_x^2 + V_y^2 + V_z^2}$

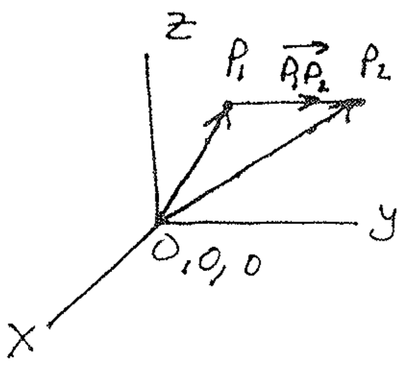


* To Calculate the vector from two points in space:

$P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$

$\overrightarrow{P_1P_2} = (x_2 - x_1)\,i + (y_2 - y_1)\,j + (z_2 - z_1)\,K$

$\overrightarrow{P_1P_2} = -\overrightarrow{P_2P_1}$



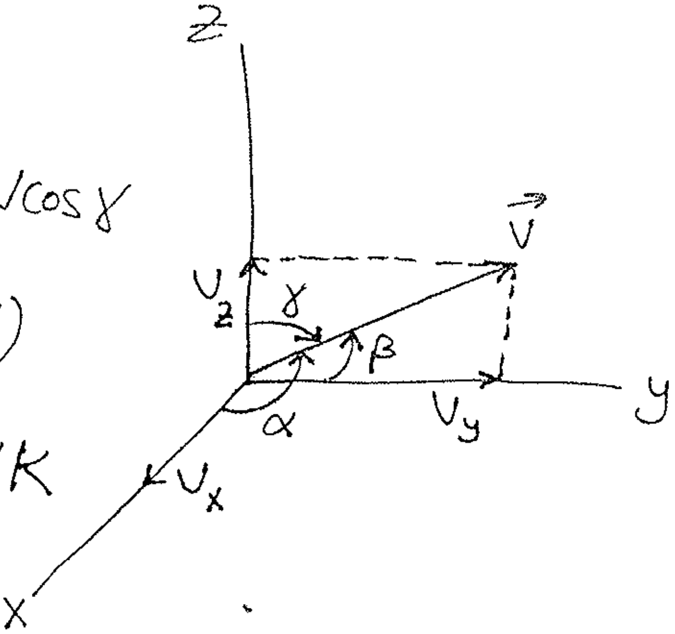
* The angles between the vectors and x-y-z axes is: α, β, γ

$\vec{V} = V_x i + V_y j + V_z K$

$V_x = V\cos\alpha, \quad V_y = V\cos\beta, \quad V_z = V\cos\gamma$

$\therefore \vec{V} = V(\cos\alpha\, i + \cos\beta\, j + \cos\gamma\, K)$

$\vec{u} = \frac{\vec{V}}{V} = \cos\alpha\, i + \cos\beta\, j + \cos\gamma\, K$



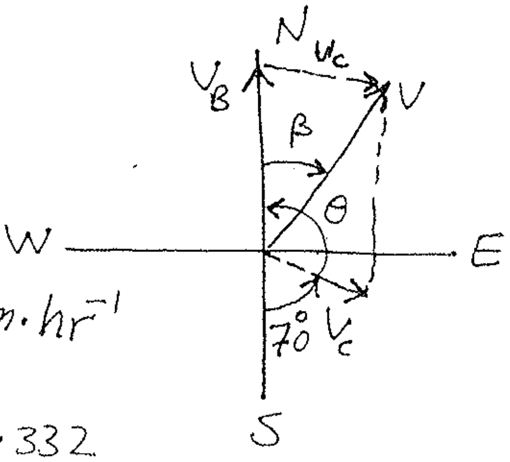
$\therefore |\vec{u}| = 1$ unit vector

$\Rightarrow \boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$ relation between the angles

ex: A motorboat is heading due north at 15 Km.hr^{-1} in a place where the current is 5 Km.hr^{-1} in the direction S 70° E. Find The resultant velocity of the boat.

sol. $\vec{V} = \vec{V_B} + \vec{V_c}$

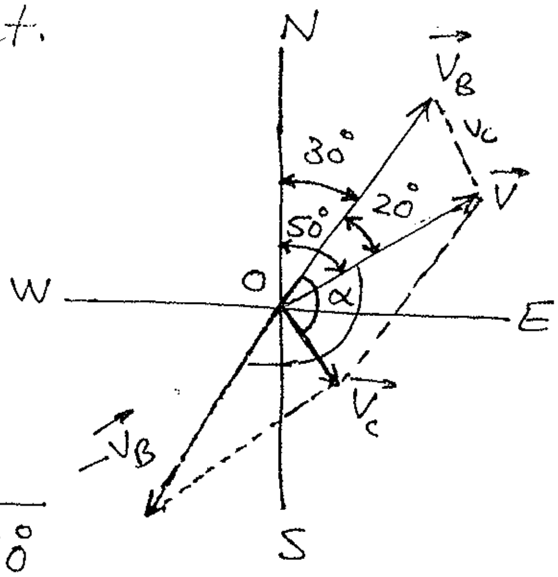
$\theta = 180° - 70° = 110°$



$V = \sqrt{15^2 + 5^2 + 2(15)(5)\cos 110} = 14.1$ Km.hr^{-1}

$\frac{V}{\sin\theta} = \frac{V_c}{\sin\beta}$ or $\sin\beta = \frac{V_c \sin\theta}{V} = 0.332$

$\Rightarrow \beta = 19.4° \Rightarrow$ N 19.4° E ex: A racing boat is heading in the direction N 30° E at $25\ Km.hr^{-1}$ in a place where the current is such that the resultant motion is $30\ Km.hr^{-1}$ in the direction N 50° E. Find the Velocity of The Current.



sol. $\vec{V} = \vec{V_B} + \vec{V_c}$

$\Rightarrow \vec{V_c} = \vec{V} - \vec{V_B}$

$V_c = \sqrt{30^2 + 25^2 - 2(30)(25)\cos 20^\circ}$

$= 10.74\ Km.hr^{-1}$.

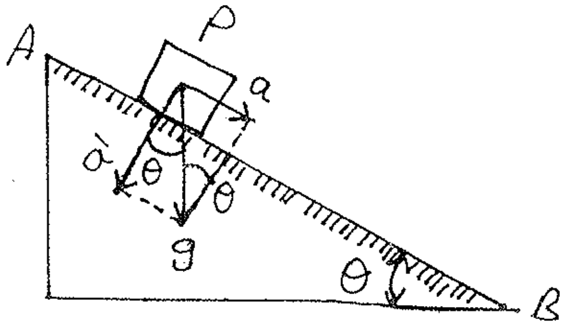
or $V_c = \sqrt{30^2 + 25^2 + 2(30)(25)\cos 160^\circ}$

$= 10.74\ Km.hr^{-1}$

The angle α between V_c and V_B: $\dfrac{V}{\sin\alpha} = \dfrac{V_c}{\sin 20^\circ} \Rightarrow \sin\alpha = \dfrac{V\sin 20}{V_c}$

$\sin\alpha = \dfrac{\overset{10.260}{30\sin 20}}{10.74} = 0.955 \Rightarrow \alpha = 72^\circ$

Note: Acceleration along an inclined plane:
سطح مائل



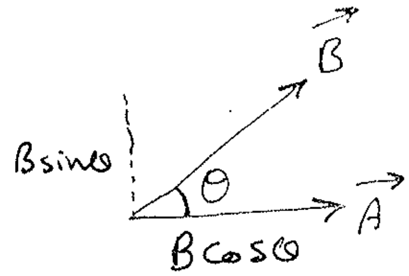
$a = g\sin\theta$

$a` = g\cos\theta$

g: gravity acceleration $= 9.8\ m.sec^{-2}$

* Scalar Product : (dot Product)

$$\boxed{\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta}$$



θ : The angle between two vectors.

$\vec{A}\cdot\vec{B} = 0 \quad (\vec{A}\perp\vec{B}) \quad \Rightarrow \theta = 90^\circ, \cos 90 = 0$

$i\cdot i = j\cdot j = K\cdot K = 1$ and $i\cdot j = j\cdot K = K\cdot i = 0$

also: $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ عملية الإبدال

and $\vec{C}\cdot(\vec{A}+\vec{B}) = \vec{C}\cdot\vec{A} + \vec{C}\cdot\vec{B}$

H.W : From scalar Product, drive the formula:

$V = \sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos\theta}$ for $\vec{V} = \vec{V_1} + \vec{V_2}$

ex: find the angle between $\vec{A}$ and $\vec{B}$ for:

$\vec{A} = 3i - j + K \quad , \quad \vec{B} = 2i + j - K$

Sol.

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{(3i-j+K)\cdot(2i+j-K)}{\sqrt{9+1+1}\ \sqrt{4+1+1}}$$

$$\cos\theta = \frac{6-1-1}{\sqrt{11}\ \sqrt{6}} = \frac{4}{\sqrt{66}} \Rightarrow \theta =$$

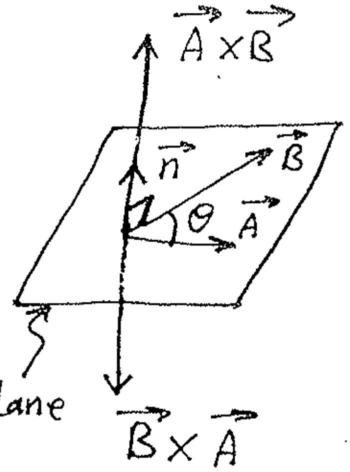
Note : If $\vec{A} = A_x i + A_y j + A_z K$ and $\vec{B} = B_x i + B_y j + B_z K$

$$\therefore \boxed{\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z}$$

اشتقاق $(A_x + A_y + A_z)\cdot(B_x + B_y + B_z)$

H.W : Find the constant a; If the vector $\vec{A} = 3i - j + 2K$ and The vector $\vec{B} = ai + 2j - K$ are Perpenduclar.

* Vector Product: (cross product)



$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta \, \vec{n}$$

where $\vec{n}$ is the unitvector perpendicular on both $\vec{A}$ and $\vec{B}$ also on plane.

$\boxed{\vec{N} = \vec{A} \times \vec{B}}$ Normal Vector on plane contain $\vec{A}$ and $\vec{B}$

$\therefore \vec{N} = \vec{n}\,|\vec{N}| \Rightarrow \boxed{\vec{n} = \dfrac{\vec{N}}{|\vec{N}|}}$

نستخدم قاعدة اليد اليمنى لتحديد
اتجاه المتجه الجديد حيث يشير
السبابة الى $\vec{A}$ والوسطى الى B
والابهام الى المتجه الجديد.

Notes: ① let $\vec{A} = A_x i + A_y j + A_z K$, $\vec{B} = B_x i + B_y j + B_z K$

$$\Rightarrow \vec{A} \times \vec{B} = \begin{vmatrix} i & j & K \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = i(A_y B_z - A_z B_y) - j(A_x B_z - A_z B_x) + K(A_x B_y - A_y B_x)$$

② ⓐ If $\vec{A} \parallel \vec{B} \Rightarrow \vec{A} \times \vec{B} = 0 \Rightarrow \theta = 0 \Rightarrow \sin\theta = 0$

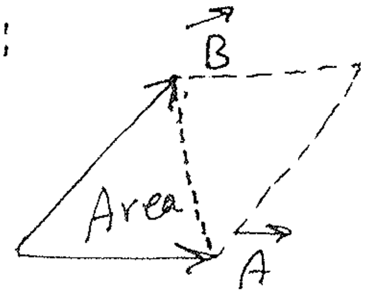
ⓑ If $A \perp B \Rightarrow A \times B = AB \Rightarrow \theta = 90° \Rightarrow \sin 90 = 1$

③ $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

④ $i \times i = j \times j = K \times K = 0$ and $i \times j = K$, $j \times K = i$, $K \times i = j$

⑤ The Area of Triangle by vectors is:

$\boxed{\text{Area} = \frac{1}{2}|\vec{A} \times \vec{B}|}$



ex: Find the Unit vector Perpendicular on $\vec{A}$ and $\vec{B}$ for:

$$\vec{A} = i - 2k \quad , \quad \vec{B} = 3i - j + 2k$$

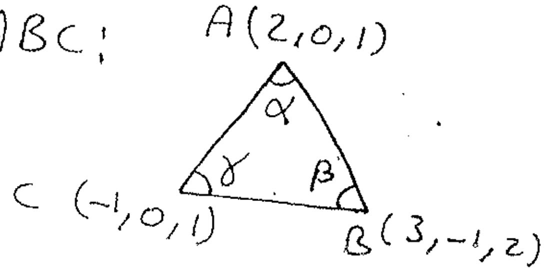
Sol:

$$\vec{N} = \vec{n}\,|\vec{N}| \Rightarrow \vec{n} = \frac{\vec{N}}{|\vec{N}|} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 1 & 0 & -2 \\ 3 & -1 & 2 \end{vmatrix} = i(0-2) - j(2+6) + k(-1-0) = -2i - 8j - k$$

$$\vec{n} = \frac{-2i - 8j - k}{\sqrt{4+64+1}} = \frac{1}{\sqrt{69}}(-2i - 8j - k)$$

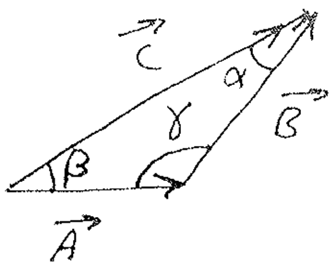
H.W (1) Find the Angles of triangle ABC:



H.W (2) Find the Area of triangle in example (H.W (1))

H.W 3: by vector Product prove the relation:

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$



problems:

Q1/ Two vectors 6 and 9 units long, from an angle of ⓐ $0°$, ⓑ $60°$ ⓒ $90°$ ⓓ $150°$ and ⓔ $180°$ Find the magnitude of their resultant and its direction with respect to shorter vector.

Q2/ Two vectors 10 and 8 units long from an angle of ⓐ $60°$ ⓑ $90°$ ⓒ $120°$ Find the magnitude of difference and the angle with respect to the Larger vector.

Q3/ Three vectors in a plane are respectively 6, 5 and 4 units long. The first and second from an angle of $50°$. while the second and third from an angle of $75°$. Find the magnitude of the resultant and its direction with respect to the larger vector.

Q4/ Given the vectors $\vec{A} = 3i + 4j$ and $\vec{B} = -i + j$ Find ① $A+B$ ② $A-B$ ③ the angle between $\vec{A}$ and $\vec{B}$.

Q5/ Given three vectors $\vec{V_1} = -i + 3j$, $\vec{V_2} = 3i - 2j$

$\vec{V_3} = 4i + 4j$ (a) Determine the products

$\vec{V_1} \times (\vec{V_1} + \vec{V_2})$ and $(\vec{V_1} + \vec{V_2}) \times \vec{V_3}$

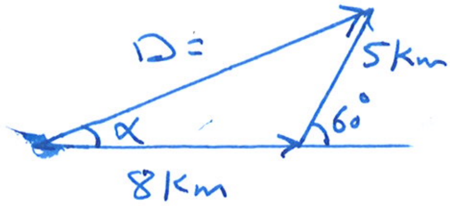
(b) Find $\vec{V_1} \cdot (\vec{V_2} \times \vec{V_3})$ and $(\vec{V_1} \times \vec{V_2}) \cdot \vec{V_3}$

Q6/ prove that the vectors $\vec{A} = 3i + 3j + 3k$

$\vec{B} = 2i + j - 3k$ are perpendicular.

problems:

Q1/ A motor boat is heading due East at (8 Km) distance and make an angle in the direction of (N60E) to reach (5 Km) and stoped. Find the resultant distance of the boat and its direction?



Sol

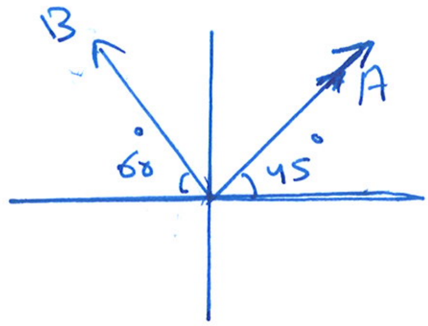
$$D = (8)^2 + (5)^2 + 2(8)(5)\cos 60$$

$$D^2 = 129\ Km^2 \Rightarrow D = 11.4\ Km$$

$$\tan\alpha = \frac{5\sin 60}{8 + 5\cos 60} = \frac{4.25}{10.25} = 0.414$$

$$\alpha = \tan^{-1}(0.414) = 22.49^\circ$$

Q2/ TWo vectors $A = 8$ cm, $B = 6$ cm as a Fig. Find the resultant and direction of A, B.



Sol نحلل كل من المتجهين A ، B إلى مركباتها العمودية والافقية نحصل على .

x.axis	y.axis
$A\cos 45$	$A\sin 45$
$-B\cos 60$	$B\sin 60$

$$R_x = A\cos 45 + (-B\cos 60)$$
$$= (8)(0.707) + (-6(0.5)) = 5.656 - 3 = 2.656\ cm$$

$$R_y = A\sin 45 + B\sin 60$$
$$= (8)(0.707) + 6(0.866) = 5.656 + 5.196 = 10.852\ cm$$

$$R^2 = R_x^2 + R_y^2 \Rightarrow R = \sqrt{R_x^2 + R_y^2} = \sqrt{7.054 + 117.76}$$
$$= \sqrt{124.8} \Rightarrow R = 11.17\ cm$$

$$\tan\alpha = \frac{R_y}{R_x} = \frac{10.852}{2.656} = 4.082 \Rightarrow \alpha = \tan^{-1}(4.082) =$$

Q3/ Given $\vec{a}=(1,-1,2)$, $\vec{b}=(2,3,-1)$, $\vec{c}=(8,7,1)$ Find:

(1) $\vec{a}-\vec{b}$, (2) $2\vec{a}+\vec{b}$, (3) $3\vec{a}-7\vec{b}$, (4) $2\vec{a}+3\vec{b}-\vec{c}$

<u>sol</u>

(1) $a-b=(1,-1,2)-(2,3,-1)=(-1,-4,3)$

(2) $$2\vec{a}+\vec{b}=2(1,-1,2)+(2,3,-1)$$
$$=(2,-2,4)+(2,3,-1)=(4,1,3)$$

(3) $$3\vec{a}-7\vec{b}=3(1,-1,2)-7(2,3,-1)$$
$$=(3,-3,6)-(14,21,-7)$$
$$=(-11,-24,13)$$

(4) $$2\vec{a}+3\vec{b}-\vec{c}=(2,-2,4)+(6,9,-3)-(8,7,1)$$
$$=(0,0,0)=0$$

Q4/ Given $\vec{a}=(1,-2,3)$, $\vec{b}=(-4,1,0)$ Find the scalar value of the vectors. (1) $|\vec{a}|$, (2) $|\vec{b}|$, (3) $|\vec{a}+\vec{b}|$ (4) $|\vec{a}-\vec{b}|$, (5) $|-7\vec{a}|$

(8) $|2\vec{a}-3\vec{b}|$.

<u>Sol</u>

$$|\vec{a}|=\sqrt{(1)^2+(-2)^2+(3)^2}=\sqrt{14}, \quad |\vec{b}|=\sqrt{(-4)^2+(1)^2+(0)^2}=\sqrt{17}$$

$$\vec{a}+\vec{b}=(-3,-1,3)$$
$$|\vec{a}+\vec{b}|=\sqrt{(-3)^2+(-1)^2+(3)^2}=\sqrt{19}$$
$$\vec{a}-\vec{b}=(5,-3,3) \Rightarrow |\vec{a}-\vec{b}|=\sqrt{(5^2)+(-3)^2+(3)^2}=\sqrt{43}$$
$$-7\vec{a}=-7(1,-2,3)=(-7,14,-21)$$
$$|-7\vec{a}|=\sqrt{(-7)^2+(14)^2+(-21)^2}=\sqrt{686}$$
$$|2\vec{a}-3\vec{b}|=|2(1,-2,3)-3(-4,1,0)|=|14,-7,6|$$
$$=\sqrt{(14)^2+(-7)^2+(6)^2}=\sqrt{281}$$